MATHEMATICAL MODEL OF INDUCTION HEATING OF A CYLINDER FOR COMPUTER-AIDED MANUFACTURING (CAM)

V. G. Kunshchikov

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The article contains computational algorithms for the parameters of induction heating of a long ferromagnetic cylinder in a longitudinal magnetic field. The algorithms can be used to determine the operating conditions of heating devices rapidly and accurately in computer-aided plastic working of metals on the basis of numerical solutions of the heat-conduction equation and the equation of electrodynamics.

One of the many advantages of induction heating applied before plastic working of metal is a high heating rate and, consequently, the possibility that the heating devices will operate at the same rate as quick-operating units such as a hammer, a press, and an edging machine. Functioning of CAM is impossible without rapid and accurate calculation of technological operating conditions of plants. Therefore, contradictory requirements regarding speed and quality of computation are sometimes imposed on computer systems for technological preparation of manufacturing. We tried to develop a fast and rather accurate computational algorithm for the thermal and electromagnetic processes that take place in heating a long ferromagnetic cylinder in a longitudinal magnetic field.

Heating of a cylinder of diameter $2R$ is described in a cylindrical coordinate system by the system of differential equations [1]

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\rho r\frac{\partial \dot{H}}{\partial r}\right)=j\omega\,\mu\,\mu_0\dot{H}\,;$$

$$c\,\frac{\partial\Theta}{\partial\tau}-\frac{1}{r}\frac{\partial}{\partial r}\left(\lambda r\frac{\partial\Theta}{\partial r}\right)=\rho\left|\frac{\partial\dot{H}}{\partial r}\right|^2, \tag{1}$$

where $\rho=\rho(\Theta)$; $j=\sqrt{-1}$; $c=c(\Theta)$; $\lambda=\lambda(\Theta)$.

The initial condition for the heat-conduction equation is as follows:

$$\Theta\,(r,\tau=0)=\Theta_0\,(r)\,. \tag{2}$$

The boundary conditions for system (1) have the form

$$\dot{H}\,(R)=\dot{H}_s\,;\ \ \frac{\partial}{\partial r}\dot{H}\,(r=0)=0\,; \tag{3}$$

$$-\lambda\frac{\partial}{\partial r}\Theta\,(R,\tau)=\Delta f\,;\ \ \frac{\partial}{\partial r}\Theta\,(r=0,\tau)=0\,. \tag{4}$$

For ferromagnetic steels the dependence of the magnetic permeability μ on the magnetic intensity was approximated by the following function:

Institute of Machine Science of the Ural Branch of the Russian Academy of Sciences, Ekaterinburg, Russia.
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$$\mu = C\,(|\dot{H}|/\sqrt{2}\,)^{(1-n)/n} \quad \text{at} \quad \Theta < \Theta_C ;$$
$$\mu = 1 \quad \text{at} \quad \Theta \geq \Theta_C . \tag{5}$$

There are literature reports about software for solution of the formulated problem. For example, in [1] the electric and thermal problem was solved by means of a matrix-factorization technique and an implicit iteration-difference scheme of representation of system (1). As was stated above, specific features of using mathematical models for CAM programs necessitates a suitable approach to development of algorithms. A difficulty consists in the need to seek reasonable compromises between the complexity and accuracy of the computational algorithms.

This problem was solved in two stages. First, a simple but rather general and accurate algorithm was developed, which was, however, not fast enough. Then, particular cases were isolated and possible simplifications made. Results of computations following the simplified algorithms were compared with those of a more general solution and then a conclusion was made about the applicability of a particular computational procedure.

In both the development of the general algorithm and the implementation of the simplified procedures, we used the method of a separate, alternate numerical solution of the heat-conduction equation and the equation of the electromagnetic field although their simultaneous solution gives more accurate results with the same discreteness of division of the time axis into intervals [1]. This allowed us to determine separately the temperature field in every time step by finding the power distribution of the heat sources from the solution of the electromagnetic problem.

The temperature field was computed rather rapidly by a fast factorization method, using an unconditionally stable implicit scheme of finite-difference approximation of the heat-conduction equation [2]. Efforts were mainly spent on a search for the ways of solutuion of the equation of electrodynamics that are given in the following.

The first equation in (1) can be written as a system of two ordinary differential equations:

$$\frac{d\dot{H}}{dr} = -\dot{J}\,, \quad \frac{d\dot{J}}{dr} + \frac{\dot{J}}{r} = -j\,\omega\,\mu\,\mu_0\dot{H}/\rho\,. \tag{6}$$

This system could be solved by a numerical method of integration of systems of differential equations, in particular, the Runge–Kutta method [3]. This would ensure simplicity, generality, and the required high performance of the algorithm. However, the numerical methods just mentioned require boundary conditions for all variables on the same boundary. In this case, conditions (3) are determined on opposite boundaries. This difficulty was overcome with the aid of an additional algorithm for seeking a maximum of the residual of one of the boundary conditions (3) in every step of numerical integration of system (6).

The algorithm obtained can be used in calculation of electromagnetic fields with very different distributions of the electromagnetic properties over the cross section of the cylinder. Tests showed almost complete coincidence of the results with literature examples of analytical and numerical calculations.

However, the need to seek boundary conditions on the same boundary resulted in a larger amount of computations. Therefore, a second stage of studies was needed that should ensure the required high performance of the computations by using adequate simplifications of the formulation of the problem and isolation of particular cases.

For calculation of heating of a nonmagnetic cylinder ($\Theta(r) \geq \Theta_C$), it is not necessary to solve the mentioned extremal problem. In this case at $\mu = 1$ system (6) is linear and homogeneous. Its general form is

$$\frac{d\mathrm{y}}{dr} = A\,(r)\,\mathrm{y}\,. \tag{7}$$

It is assumed that a solution of this system $y_1(r)$ that satisfies the second boundary condition in (3) is known. Because of the linearity and homogeneity of (7), any solution of the form $y(r) = py_1(r)$ is its solution. It can easily be seen that this solution ensures satisfaction of the second condition in (3). The constant p can be determined from the first boundary condition. The approach described is implemented in practice in a numerical calculation as follows. Any $\dot{H}(r = 0)$ is assigned. Since $\dot{J}(r = 0)$ is specified by the boundary condition, all the

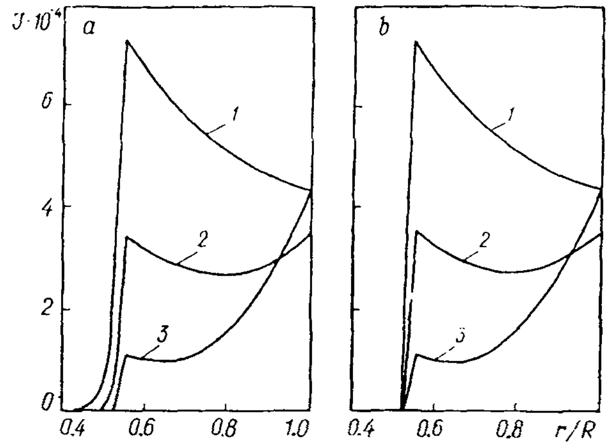

Fig. 1. Distribution of the current density J, A/m^2, over the radius of a cylinder: a) general algorithm; 2) simplified algorithm: 1) $m = 2$; 2) 4; 3) 8.

variables in (7) are determined on the boundary $r = 0$, and numerical integration can result in the mentioned solution $y_1(r)$. Determination of the constant p leads to the desired result.

The following particular case appears in heating of a cylinder that is ferromagnetic over the whole cross section, when $\Theta(r) < \Theta_C$. Here a strong surface effect occurs. It leads to the fact that the effective depth of penetration of current constitutes fractions of a millimeter for a wide range of current frequency [4]. For this case numerical calculation of the electromagnetic field requires a very fine division of the spatial coordinates and a consequent increase in the amount of computation. Together with the need to solve an extremal problem in every step of integration of system (6), this factor makes the computational algorithm described above unacceptable for CAM, although it permits finding the electromagnetic field with high accuracy for almost any conditions heating.

Neiman's method [5] can reduce substantially the amount of computation. With such a strong surface effect, the method developed for the case of a plane electromagnetic wave is also quite suitable for cylindrical systems. The essence of this method consists in replacement of the relation $\mu = \mu(|\dot{H}|)$ by a relation $\mu = \mu(r)$ of the form

$$\mu(r) = \frac{\mu(R)}{\left(1 - \frac{R - r}{r - r_0}\right)^2}. \tag{8}$$

Using (8), it is possible to obtain simple formulas for calculation of the parameters of the electromagnetic field at $\Theta < \Theta_C$ [5].

Thus, heating of a ferromagnetic cylinder and a cylinder that lost its ferromagnetic properties can be calculated with a minimum amount of computation without numerical minimization of the residual of the boundary conditions on the surface. The case in which the surface layers are heated above the Curie point and the bulk layers are still ferromagnetic (i.e., the case of a cylinder consisting of layers with different magnetic properties) has not yet been considered.

If at the boundary of the layers at $\Theta = \Theta_C$ and $r = r_1$ the absolute value of the magnetic intensity $|\dot{H}(r)_1|$ were known, Neiman's method could be used for the bulk, cold part of the cylinder. An artificial means was found that reduces substantially the amount of computation for this case too. It was assumed (and the validity of this assumption was proved subsequently) that the magnetic permeability at the boundary of the layers $\mu(r_1)$ depends linearly on the thickness of the heated layer

$$\mu(r_1) = \mu(|\dot{H}_s|) + \nu(R - r_1), \tag{9}$$

where $\mu(|\dot{H}_s|)$ is determined from (5) for $|\dot{H}| = |\dot{H}_s|$; ν is a coefficient assigned from the condition of maximum similarity of results of the simplified and general solutions. With this replacement, it is possible to use Neiman's

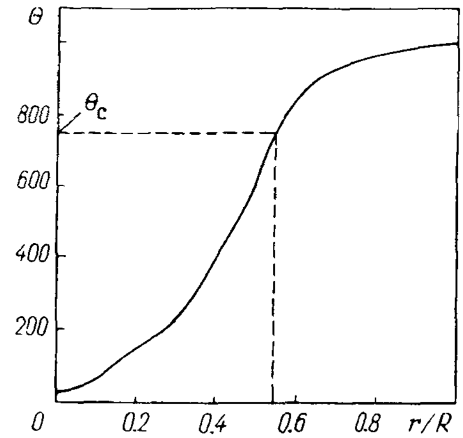

Fig. 2. Distribution of the temperature Θ, °C, over the radius of a cylinder.

method for the central ferromagnetic part of the cylinder and to reduce system (6) to the form (7). Here, for the nonmagnetic part of the cylinder, calculation is carried out in a way similar to the case of a cylinder completely heated above the temperature of magnetic transformations.

As an example, the distribution of the absolute value of the current density $J = |\dot{J}|$ in a steel cylinder is shown in Fig. 1. The distribution is found following the general and simplified algorithms, respectively, for $|\dot{H}_s|$ = 50,000 A/m for different values of the dimensionless parameter $m = R\sqrt{\omega\mu_0/\rho(\Theta)_C}$, where $\rho(\Theta_C) = 10^6\ \Omega\cdot\text{m}$ is taken. The temperature field, shown in Fig. 2, provides magnetic properties of the cylinder that are different in the two layers. The layer that lies to the right of the dashed line in Fig. 2 is paramagnetic, and that located to the left of this line is ferromagnetic. It can be seen that the simplified algorithm practically does not lead to a loss of computational accuracy. Meanwhile, the speed of computation on a personal computer of the IBM/PC type absolutely ensures that this algorithm could be used in CAM.

On the basis of the procedure described, a CAD system was developed for induction heating of cylindrical steel blanks [6]. Using this system, it is possible to find the voltage on the inductor that provides the required temperature distribution over the cross section of the blanks. The system also allows determination of the most effective heating conditions with limitations imposed on the process parameters.

Thus, computational algorithms for the temperature field in induction heating of a ferromagnetic cylinder are developed and implemented in the form of computer programs. A general algorithm based on numerical solution of a system of differential equations of electrodynamics with minimization of the residual of one of the boundary conditions can be used to determine the electromagnetic field in the cylinder for almost any distribution of the electromagnetic properties over the radius of the blank.

The simplified algorithms that describe the process of induction heating of a cylinder in particular practical cases allow computations with a speed suitable for CAM without losses of accuracy that are important in practice.

NOTATION

R, radius of the cylinder; ρ, specific electrical resistance; $\dot{H}$, complex magnetic intensity; ω, angular frequency of the current in the winding of the inductor; μ, relative magnetic permeability; μ_0, permeability of vacuum; r, radial coordinate; Θ, temperature; c, specific heat capacity per unit volume; λ, thermal conductivity; τ, time; $\Theta_0(r)$, initial radial distribution of the temperature of the cylinder at $\tau = 0$; $\dot{H}_s = \dot{H}(R)$, magnetic intensity on the surface of the cylinder; Δf, specific power of heat losses from the lateral surface of the cylinder; C and n, coefficients depending on the steel grade; Θ_C, temperature at the point of magnetic transformation (Curie point); $\dot{J}$, complex current density; $y = (\dot{H}, \dot{J})$, vector; $A(r)$, matrix-valued function; $y_1(r)$, particular solution of system (7); p, constant; r_0, value of r for the case of an almost completely absent electromagnetic field; r_1, value of r at which $\Theta = \Theta_C$.

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